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# A statistical spectroscopy approach for calculating nuclear level densities

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**Abstract.** We compute level densities with a nuclear statistical spectroscopy approach. This model is based on the microscopic physics of the interacting shell model. The level density is constructed by means of a sum of binomial distributions. The partial densities correspond to different configurations of the valence space. The individual binomial parameters fit the exact configuration moments. Calculations of level densities for *sd*-shell nuclides show the model to work well, when compared to exact calculations with full-diagonalization shell model.

**PACS.** 21.10.-k Properties of nuclei; nuclear energy levels – 21.10.Ma Level density – 26.50.+x Nuclear physics aspects of novae, supernovae, and other explosive environments

## 1 Introduction

The level density —number of excited states per MeV of excitation energy— is an important input to estimate reaction rates in nucleosynthesis. Theoretical prescriptions to calculate nuclear level densities have been developed since Bethe's formulation [1]. Modifications to this model have been made to include important nuclear properties such as shell effects or neutron resonance spacings [2,3,4]. On the other hand, full shell model calculations are now possible up to $A \approx 70$, although these are often limited by heavy computational loads. We use statistical spectroscopy concepts [5] to model the level density in the shell model using the binomial distribution, first suggested by Zuker [6]. We use the configuration scheme —$m$-particles distribuited over spherical shell model $j$-orbits— to compute level densities. The purpose of this work is not to compute "exact" shell model calculations [7], but to use binomial distributions to model the configuration level densities and get the approximate secular behavior of the total level density. We apply the binomial fit to each configuration and sum over all of them to obtain the total level density. The exploration of the model is currently done in *sd*-shell nuclei. A brief description of the method is presented, and some preliminary results explained.

## 2 The method

Statistical spectroscopy argues that the nuclear properties are defined by the nuclear moments. Following this approach we get the configuration moments of the nucleus and construct a binomial fit for each partition of the shell model space. We then get the *total level density* by adding all the partial densities. The sum of the partitioned binomials (SUPARB) can be compared then, at least for light nuclei in the *sd*-shell, against full-diagonalization shell model calculations.

### 2.1 The binomial distribution

Every configuration of the valence space contributes with a *partial density*. Based on the lowest nuclear moments, each partial density is constructed with a binomial fit

$$(1+\lambda) = \sum_{k=0}^{N} \lambda^k \binom{N}{k}, \qquad (1)$$

where $N$ and $\lambda$ are obtained from conditions to fit the width and third moment of a given configuration, and $k$ is related to the scaled excitation energy.

We make use of analytic formulae to compute the moments [8,9]. The point of using this procedure is to avoid a full shell model calculation to get the configuration moments. Even when the computation of the moments using such formulae is computationally exhaustive for heavier nuclei, it is still a lot more convenient than performing a full shell model calculation. Moreover, the partitioned scheme we use is highly suitable to run on parallel computers and substantially reduce the time to calculate the moments. Once we have the binomial fit for every configuration we get the total level density straightforwardly by

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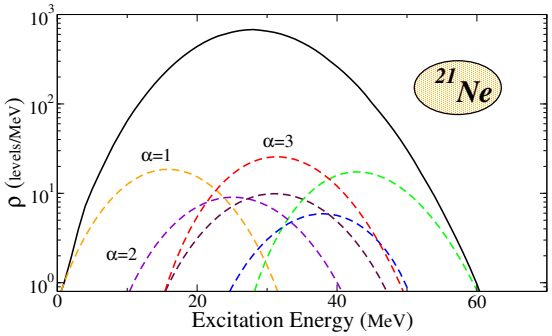


**Fig. 1.** Calculated level density for $^{21}$Ne. A few individual contributions $\rho_\alpha$ (broken lines, coloured on line) are shown. The total level density (solid, black line) is the sum over all the configurations. The unscaled Wildenthal interaction was used.

adding up the calculated partial densities. To illustrate this, a qualitative plot of the configurations densities is shown in fig. 1. The plots of broken lines in the figure correspond to a selected set of partial densities, labeled by $\alpha$. In the case of $^{21}$Ne there are only 54 of such configurations. One advantage of this method is that it is mostly analytic, so it will be suitable for nuclei in the *sdpf*-shell, which have thousands of valence space configurations (*i.e.* $5 \times 10^5$ for $^{54}$Fe up to 3p-3h configurations).

### 2.2 Performance of the model

To test the reliability of the method, we have performed calculations of level densities for several nuclides in the *sd*-shell with the Wildenthal interaction [10] and compared them to exact calculations. To accomplish the comparison we made use of the REDSTICK shell model code [11]. Figure 2 shows the $^{20}$Ne, $^{34}$Ar and $^{34}$Cl level densities computed with the SUPARB method described here, compared against exact shell model calculations. The SUPARB computations shown in fig. 2 reveal an overall good fit of the exact level density in all three cases, although our interest is in the low-energy region (up to 20 MeV), where experimental data is available. We get similar results for other *sd*-shell nuclei. We plan to extend these calculations to larger spaces, namely, the *sdpf* and $sdpf + g_{9/2}$ shells, where exact calculations are very difficult to perform.

Besides the level density, computations of locally averaged expectation values of operators and $J$-dependent level densities with spin cut-off are also possible within this approach. We will explore these possibilites in the short term.

## 3 Conclusions

The SUPARB method to calculate level densities has been described here. The valence space is partitioned and the total level density obtained from the sum of the corresponding partial densities. In our preliminary calculations we have seen evidence that the binomial distribution fits well the level density for *sd*-shell nuclei, when compared against full-diagonalization shell model calculations. Besides this, the method presented here has also shown to have the advantage of being computationally cheap, since we use analytic formulae and implement the calculation of the moments in parallel computers.

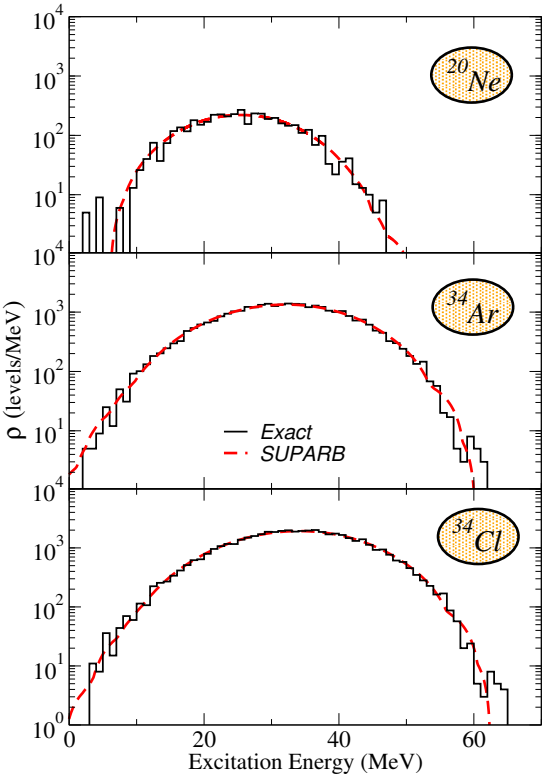


**Fig. 2.** $^{20}$Ne, $^{34}$Ar and $^{34}$Cl level densities in the *sd*-shell. The SUPARB calculation (broken line) described here is compared with the "exact" shell model calculations [11] (solid, stair line).

We will continue the study of light nuclei, although our ultimate goal is to compute level densities in the *pf*-shell, namely, V, Mn, Ti, Co isotopes. Also, we are interested in testing different interactions like the Gogny interaction or the surface-delta interaction to see how the results depend on the matrix elements used. Furthermore, we have a particular interest to see how the computed level density with this method compares to available experimental data in the *pf*-shell. These investigations are currently underway.

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